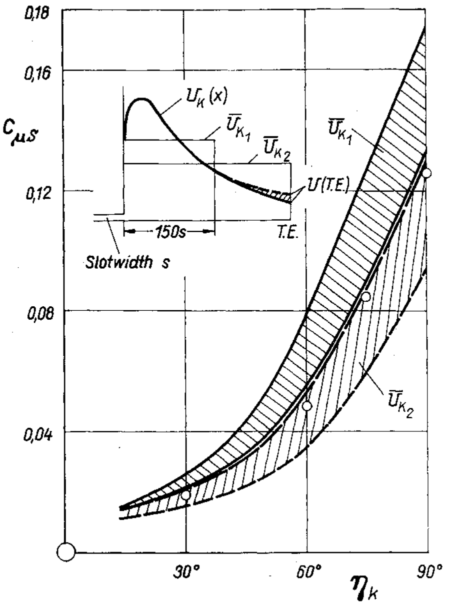

Fig. 2 Momentum coefficient $c_{\mu S}$ for preventing separation from experiment and theory. Slot width s/c = 0.8 × 10^{-3}, ○ = experiment, —— = theory with $\bar{U}_K = \bar{U}_{K1}$, and --- = theory with $\bar{U}_K = \bar{U}_{K2}$. Hatched area represents scatter due to uncertainty of the value of U at the trailing edge.

ing. This is achieved when the net amount of the blown-in jet momentum $\eta_\vartheta \vartheta_i$ is equal to the increase in the momentum thickness ϑ_G of the boundary layer with no blowing which occurs between the separation point S and the trailing edge (see Fig. 1c). Thus,

$$-\eta_\vartheta \vartheta_{iS} = \vartheta_G \tag{4}$$

The increase in momentum thickness ϑ_G can easily be obtained by calculating the turbulent boundary layer as described in Ref. 5:

$$\frac{\vartheta_G}{c} = \frac{0{,}037}{Re^{1/5}} \left[\frac{U(TE)}{U_\infty}\right]^{-3} \left\{\int_S^{TE} \left[\frac{U(x)}{U_\infty}\right]^{3.5} d\left(\frac{x}{c}\right)\right\}^{0.8} \tag{5}$$

The expression for the minimum momentum coefficient required for preventing boundary-layer separation is finally found by inserting Eqs. (2) and (3) into Eq. (4):

$$c_{\mu S} = 2\frac{\vartheta_G}{c} \frac{1}{0.85\,[1 - (U_\infty/v_i)]^2} \tag{6}$$

with ϑ_G according to Eq. (5). This equation indicates that the required momentum coefficient $c_{\mu S}$ decreases with increasing jet velocity and with decreasing slot width. This is in agreement with the experimental results of different authors which are compared in Refs. 2 and 6.

For the calculation of the flapped wing in Eq. (6), it is reasonable to use the mean velocity $\bar{U}_K$ on the flap (see Fig. 1b) instead of the mainstream velocity U_∞. Thus

$$c_{\mu S} = 2\frac{\vartheta_G}{c} \frac{1}{0.85\,[1 - (\bar{U}_K/v_i)]^2} \tag{7}$$

By this method, which is described in more detail in Ref. 2, a good agreement with measurements has been achieved. A comparison between experiment and theory is given in Fig. 2. The hatched areas indicate how the uncertainty in the calculation of the velocity at the trailing edge $U(TE)$ and the definition of the mean value $\bar{U}_K$ influence the results. This method has also been extended to airfoils using boundary-layer control by blowing at the leading edge.[7]

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Flow of Viscoelastic Maxwell Fluid in a Circular Pipe

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RICHARDSON and Tyler[1] investigated experimentally the flow of a Newtonian, viscous fluid in a circular tube under a periodic pressure gradient, and Sexl[2] investigated the same problem theoretically. Sanyal[3] discussed the same problem under a pressure gradient, rising as well as falling exponentially with time. The object of this note is to study the flow of viscoelastic Maxwell fluid in a tube of circular section, following Sanyal, in two cases: 1) when the pressure gradient rises exponentially with time, and 2) when the pressure gradient falls exponentially with time.

Following Bagchi,[4] the equations of motion are

$$\left(1 + \lambda\frac{\partial}{\partial t}\right)\frac{\partial w}{\partial t} = -\frac{1}{\rho}\left(1 + \lambda\frac{\partial}{\partial t}\right)\frac{\partial p}{\partial z} + \nu\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) \tag{1a}$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial r} \tag{1b}$$

where

$$\nu = \mu/\rho \tag{1c}$$

Equation (1b) shows that pressure p is a function of z and t only.

Case I: Pressure Gradient Rising Exponentially with Time

Let us assume for the pressure gradient

$$-(1/\rho)(\partial p/\partial z) = ke^{\alpha^2 t} \tag{2a}$$

and for the velocity

$$w = f(r)e^{\alpha^2 t} \tag{2b}$$

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Then from Eqs. (1a, 2a, and 2b) we have

$$f'' + \frac{1}{r}f' - \frac{(1+\lambda\alpha^2)\alpha^2}{\nu}\cdot f = -\frac{k}{\nu}(1+\lambda\alpha^2) \quad (3)$$

The solution of this equation, which does not become infinite on the axis of the tube, is

$$f = (k/\alpha^2) + AJ_0(i\beta r) \quad (4)$$

where

$$\beta^2 = (1 + \lambda\alpha^2/\nu)\alpha^2 \quad (5)$$

and J_0 is a Bessel function of order zero, of imaginary argument $i\beta r$, and A is the constant of integration. Then upon determining A from the boundary condition

$$r = R \qquad w = f = 0 \quad (6)$$

where R is the radius of the tube, we get

$$f = \frac{k}{\alpha^2}\left(1 - \frac{J_0(i\beta r)}{J_0(i\beta R)}\right) \quad (7)$$

and

$$w = \frac{k}{\alpha^2}\left(1 - \frac{J_0(i\beta r)}{J_0(i\beta R)}\right)e^{\alpha^2 t} \quad (8)$$

Another quantity of interest from the engineering point of view is the amplitude ϕ of mass flow, which is given by

$$\phi = 2\pi\rho\int_0^R frdr = \frac{\pi\rho kR^2}{\alpha^2}\left(1 - \frac{2J_1(i\beta R)}{i\beta RJ_0(i\beta R)}\right) \quad (9)$$

We now work out these results by expanding the Bessel functions for small values of $|\beta R|$ and $|\beta r|$ by using the formulas

$$J_0(z) = 1 - \tfrac{1}{4}z^2 + (\tfrac{1}{64})z^4 \ldots$$
$$J_1(z) = \tfrac{1}{2}(z) - (\tfrac{1}{16})z^3 + \ldots \quad (10)$$

Equations (8) and (9), in virtue of (10), become, upon neglecting terms higher than β^2,

$$w = \frac{k\beta^2}{4\alpha^2}(R^2 - r^2)e^{\alpha^2 t} = \frac{\beta^2}{4\alpha^2}(R^2 - r^2)\left(-\frac{1}{\rho}\frac{\partial p}{\partial z}\right) \quad (11)$$

$$\phi = (\pi\rho kR^4/4\alpha^2)\beta^2 \quad (12)$$

From Eq. (11), it can be seen that, if β is constant, i.e., λ is constant, the velocity at a given point increases exponentially with time. The distribution of velocity in a cross section of the tube is parabolic, as in the Hagen-Poiseuille flow, and the velocity varies in the same phase with that of pressure gradient.

We now work out results by expanding Bessel functions for large values of $|\beta R|$ and $|\beta r|$ by using the formula

$$J_m(z) = (2/\pi z)^{1/2}\cos[z - (m\pi/2) - (\pi/4)] \quad (13)$$

Equations (8) and (9), in virtue of (13), become, respectively,

$$w = (k/\alpha^2)[1 - (R/r)^{1/2}\cdot e^{-\beta(R-r)}]\,e^{\alpha^2 t} \quad (14)$$

$$\phi = \frac{\pi\rho kR^2}{\alpha^2}\left[1 - \frac{2}{i\beta R}\tan\left(i\beta R - \frac{\pi}{4}\right)\right] \quad (15)$$

It is important to note from Eq. (14) that the velocity increases exponentially with time at a given point. Moreover, for appreciable value of $\beta(R - r)$, where β is constant, $e^{-\beta(R-r)}$ is very small, so that the velocity is independent of the distance from the wall. Thus, only for small values of $\beta(R - r)$ does the velocity depend upon the distance from the wall. Hence the solution has the boundary-layer character even in viscoelastic Maxwell fluids, provided that β, i.e., the relaxation time λ, is constant. When λ is zero, the case reduces to that of ordinary fluids, considered by Sanyal.

Case II: Pressure Gradient Falling Exponentially with Time

Here we assume that the pressure gradient is given by

$$-(1/\rho)(\partial p/\partial z) = ke^{-\alpha^2 t} \quad (16a)$$

and the velocity is given by

$$w = f(r)e^{-\alpha^2 t} \quad (16b)$$

Then from Eqs. (1a, 16a, and 16b) we get

$$f'' + \frac{1}{r}f' + \left(\frac{1 - \lambda\alpha^2}{\nu}\right)\alpha^2\cdot f = -\frac{k(1 - \lambda\alpha^2)}{\nu} \quad (17)$$

The solution of this equation, which does not vanish on the axis, in virtue of the boundary conditions (6), is

$$f = -(k/\alpha^2)\{1 - [J_0(\beta_1 r)/J_0(\beta_1 R)]\} \quad (18)$$

Hence

$$w = -k/\alpha^2\{1 - [J_0(\beta_1 r)/J_0(\beta_1 R)]\}e^{-\alpha^2 t} \quad (19)$$

where

$$\beta_1 = (1 - \lambda\alpha^2/\nu)\alpha^2 \quad (20)$$

The amplitude is given by

$$\phi_1 = -(\pi\rho kR^2/\alpha^2)\{1 - [2J_1(\beta_1 R)/\beta_1 RJ_0(\beta_1 R)]\} \quad (21)$$

Hence, for small values of $|\beta_1 R|$ and $|\beta_1 r|$, (19) and (21) become, respectively,

$$w = (k\beta_1^2/4\alpha^2)(R^2 - r^2)e^{-\alpha^2 t} = (\beta_1^2/4\alpha^2)(R^2 - r^2)[-(1/\rho)(\partial p/\partial z)] \quad (22)$$

$$\phi_1 = (\pi\rho kR^4/4\alpha^2)\beta_1^2 \quad (23)$$

From Eq. (22) we conclude that the distribution of the velocity in a cross section of the tube is parabolic, and for a constant value of β_1, the velocity is in phase with the pressure gradient.

Again, for large values of $|\beta_1 R|$ and $|\beta_1 r|$, (19) and (21) become, respectively,

$$w = -\frac{k}{\alpha^2}\left[1 - \left(\frac{R}{r}\right)^{1/2}\cdot\frac{\cos\{(\beta_1 r) - (\pi/4)\}}{\cos\{\beta_1 R - (\pi/4)\}}\right]e^{-\alpha^2 t} \quad (24)$$

$$\phi_1 = -\frac{\pi\rho kR^2}{\alpha^2}\left[1 - \frac{2\sin[\beta_1 R - (\pi/4)]}{\beta_1 R\cos[\beta_1 R - (\pi/4)]}\right] \quad (25)$$

From Eq. (24), one can conclude that the distribution of velocity depends upon the wall distance, and hence the solution has no boundary-layer character.

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